Electron density stratification in two-dimensional structures tuned by electric field

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A new kinetic instability which results in formation of charge density waves is proposed. A spatial period of arising space-charge and field configuration is inversely proportional to electric field and can be tuned by applied voltage. The instability has no interpretation in the framework of traditional hydrodynamic approach, since it arises from modulation of an electron distribution function both in coordinate and energy spaces. The phenomenon can be observed in thin 2D nanostructures at relatively low electron density.

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Recent progress in microelectronics is related to great success in controlled fabrication of low-dimensional semiconductor systems. That is why transport properties of semiconductor nanostructures, both of classical and quantum nature, lately attract wide attention. In this paper we discuss a new type of purely classical instability, which can be observed in 2D nanostructures. The instability results in a formation of charge density waves (CDW). The main feature of the phenomenon is its kinetic nature. In contrast to usual current and density instabilities in semiconductors [1, 2, 3], the electron dynamics in the kinetic instability can not be described on the basis of the local hydrodynamic parameters, such as electronic density, drift velocity, and temperature. Remarkably, the kinetic instability can develop even in the Ohmic regime, when the stationary value of electric current is proportional to applied voltage.

Closely related phenomena of the striations formation are widely known in the gas discharge physics. The striated discharge has been observed since M. Faraday and is regarded as one of the most typical discharge forms [4, 5, 6, 7, 8]. In spite of this, a consistent theory of striation is up to now absent. In the last decades it was realized that for description of typical striated discharges the fluid approach fails. It was shown that the hydrodynamic description is valid only for very high electron densities when the collisions between electrons are frequent enough for the maxwellization of electron distribution function (EDF) [9, 10]. At lower electron densities, occurring in typical gas discharges, the phenomenon is of essentially kinetic nature. In this case, the EDF perturbation in the striations is varying both in space, and along the energy axis [11, 12, 13, 14, 15, 16], and it is impossible to parameterize it in terms of perturbations of electron density and temperature. The kinetic striations mechanism was analyzed first in [11, 12, 13]. In [11] it was argued, that the necessary conditions for kinetic stratification are: a) The momentum relaxation is much faster than the energy relaxation; b) The energy relaxation is mostly controlled by the energy gain in the external field F_0 and strong inelastic collisions with a large fixed energy transfer W_0; c) There should exist a mechanism of a weak continuous energy loss. In a spatially modulated potential $U(z) = -F_0 z + \delta U(z)$, $\delta U(z) = \delta U(z + L)$, these conditions provide [11, 12, 13] for the resonant EDF response at $L = L_0/m$, where $L_0 = W_0/F_0$, $m = 1, 2, ...$ This "resonant" behavior corresponds to the widely known empirical Novak's rule [6, 7, 8, 17]. An idea was put forward [11] that under the conditions a)-c) the instability develops which results in the formation of CDW with the same periods L_0/m. In [11, 12, 13], the electron kinetics was analyzed only in a given fixed electric potential profile $U(z)$. However, a complete analysis of instability requires self-consistent calculation of the potential perturbation $\delta U(z, t)$ in terms of the carrier densities perturbations. Since the discharge field depends crucially on the ion motion and on the complex ion generation processes, even a linear instability problem for the gas discharge plasma is still lacking a self-consistent solution.

In this paper we will demonstrate that, in principle, the kinetic stratification is also observable in low-dimensional semiconductor structures [18]. Moreover, it turns out that for the semiconductors a relatively simple self-consistent analytical solution can be found. The main simplification follows from the fact, that, in contrast to the gas discharge, a compensating positive charge is fixed and homogeneous. The stratification conditions a), b), and c) can be easily achieved in semiconductors. The momentum relaxation is usually fast compared to energy relaxation. The requirements b) and c) are also usually satisfied, the scattering by optical phonons with energy W_0 and scattering by acoustic phonons working as strong inelastic and weak quasielastic energy relaxation mechanisms. We will show that effect can be observed in 2D quantum wells with small thickness. Spatial periods of arising CDW equal to L_0/m and can be tuned by applied voltage. We assume that lattice temperature T_0, as well as Fermi energy, are small compared to W_0 (in what follows for simplicity we put $T_0 = 0$). The condition b) requires that electrons be "hot", and their energies be of the order of W_0, i.e. the electron gas in this case is non-degenerate. We also assume that electron concentration is small and neglect electron-electron collisions.

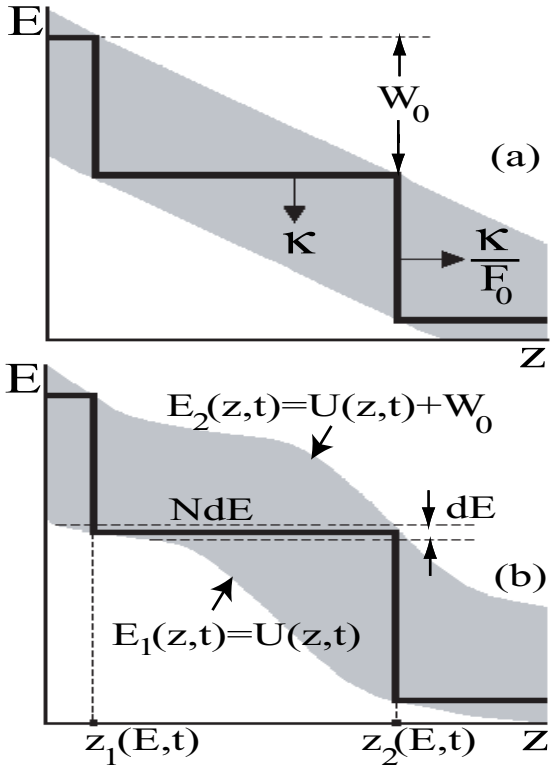

FIG. 1: (a) Motion of an electron in homogeneous field, $U_0(z) = -F_0 z$. The diffusive "staircase" trajectories slowly drift in z direction with velocity κ/F_0. (b) Motion in the modulated potential $U(z,t) = -F_0 z + \delta U(z,t)$.

Let us consider the motion of an electron in an external field F_0 assuming for a moment that the only scattering mechanism is elastic scattering. This leads to a diffusion in coordinate space. As far as energy relaxation processes are "turned off", the electron is infinitely heated by the field F_0 diffusing over the kinetic energy W. Evidently, this diffusion is strictly correlated with the diffusion in the coordinate space, since the full electron energy $E = W - F_0 z$ is conserved. In fact, an electron diffuses in (z, W) space along the line $E = const$. Now, if we take into account sufficiently intensive optical phonon emission, the electron motion will be restricted by a shell $0 < W < W_0$. In the process of diffusion with a constant total energy E, the kinetic energy increases. Reaching the point $W = W_0$, electron loses the energy W_0 and starts a diffusive motion with a lower total energy $E - W_0$. A trajectory of the electron in the space (E, z) is shown on Fig. 1a. The interaction with acoustic phonons plays a role of a friction force leading to the continuous loss of the electron energy with the rate κ due to spontaneous emission (since we have assumed that $T_0 = 0$). Later we will show that for a 2D system the rate of energy loss κ does not depend on the kinetic energy. There are three different time scales in our problem: a transport scattering time τ, a characteristic time of electron heating by the electric field $\tau_0 \sim L_0^2/D_0$ (here $L_0 = W_0/F_0$, $D_0 = W_0\tau/M$ and M is the electron effective mass), and a time W_0/κ which characterizes a rate of energy loss due to the emission of acoustic phonons. We will assume that

$$\tau \ll \tau_0 \ll \frac{W_0}{\kappa}. \tag{1}$$

Inequality (1) provides that the acoustic phonon scattering may be considered as a small perturbation. Due to this scattering the "staircase" diffusive trajectories move slowly down (along axis E) with velocity κ. This means at the same time that the trajectories slowly drift with the velocity $s_0 = \kappa/F_0$ along axis z (see Fig. 1a). Since s_0 is inversely proportional to the applied field one can say that the motion of trajectories demonstrates a negative differential mobility. It is well known that the negative differential mobility should lead to the current instability [1, 2]. However, our case is more complicated than the usual Gann instability, because one has to follow the motion of diffusive trajectories instead of the motion of individual electrons. As for latter ones, their average drift velocity obeys the usual Ohm's law $v = F_0\tau/M$ and is much larger than s_0 (since the inequalities (1) may be rewritten as $s_0 \ll v \ll \sqrt{W_0/M}$). The fact that the instability can be observed in the Ohmic regime indicates that the effect is purely kinetic and can not be described in terms of hydrodynamic parameters.

As far as the elastic collisions are dominant (see Eq.(1)), the EDF is almost isotropic [19], $f(z, W, \varphi, t) \approx f_i(z, W, t) + f_a(z, W, t)\cos(\varphi)$. Here f_i is an isotropic part of EDF, $f_a \cos(\varphi)$ is a small anisotropic correction, and φ is an angle between electron velocity and applied field. Denote $J(z, W, t) = \sqrt{W/4M} f_a$. The equations for f_i and J can be written as follows [1, 2, 19]

$$J = -D(W)\left(\frac{\partial f_i}{\partial z} + F\frac{\partial f_i}{\partial W}\right), \tag{2}$$

$$\frac{\partial f_i}{\partial t} + \frac{\partial J}{\partial z} + F\frac{\partial J}{\partial W} = \kappa\frac{\partial f_i}{\partial W}, \tag{3}$$

where $D(W) = W\tau/M$ is an energy dependent diffusion coefficient. For simplicity we assume that τ is energy independent and, consequently, $D(W)$ is proportional to W. The boundary conditions for Eqs. (2), (3) read

$$f_i|_{W=W_0} = 0, \qquad (FJ - \kappa f_i)|_{W=0} = FJ|_{W=W_0}. \tag{4}$$

Here $F(z,t) = -\partial U(z,t)/\partial z$, and $U(z,t)$ is a potential energy, which includes both the self-consistent potential created by electrons and the external potential $U_0(z) = -F_0 z$. Condition $f_i|_{W=W_0} = 0$ corresponds to the limit of a very strong interaction with optical phonons ("black wall" condition). The second boundary condition is related to the conservation of the number of particles

in inelastic collisions [20]. Eqs. (2), (3), and (4) at $\kappa = 0$ have a homogeneous stationary solution

$$J = J_0 = \frac{n_0 v}{W_0}, \quad f_i = f_{i0} = \frac{n_0}{W_0} \ln\left(\frac{W_0}{W}\right), \qquad (5)$$

where n_0 is the stationary electron concentration (we assume the following normalization $\int_0^{W_0} f_{i0} dW = n_0$). Since $\int_0^{W_0} J_0 dW = n_0 v$, solution (5) corresponds to the Ohmic regime. According to [11] we rewrite Eqs. (2), (3) in variables (E, z, t), where $E = W + U(z,t)$ is a full energy of a particle. The result is given by

$$J = -D(E-U)\frac{\partial f_i}{\partial z}, \qquad (6)$$

$$\frac{\partial f_i}{\partial t} + \frac{\partial J}{\partial z} = (\kappa - \frac{\partial U}{\partial t})\frac{\partial f_i}{\partial E}. \qquad (7)$$

The motion of a particle in the space of new variables is restricted by the curves $E = E_1(z,t) = U(z,t)$, $E = E_2(z,t) = W_0 + U(z,t)$ (See Fig. 1b). The boundary conditions (4) can be rewritten as

$$f_i|_{E=E_2(z,t)} = 0,$$
$$(FJ - \kappa f_i)|_{E=E_1(z,t)} = FJ|_{E=E_2(z,t)}. \qquad (8)$$

Since the total energy of an electron changes slowly (with the characteristic time W_0/κ), it will be useful to introduce an electron density distribution over the E axis

$$N(E,t) = \int_{z_1(E,t)}^{z_2(E,t)} dz f_i, \qquad (9)$$

where $z_1(E,t)$, $z_2(E,t)$ are the inverse functions of $E_1(z,t)$, $E_2(z,t)$, correspondingly, and the value NdE represents the number of electrons on "staircase" trajectories restricted by E and $E + dE$ (see Fig. 1b). The stationary value of N is given by $N_0 = n_0/F_0$. Introducing the notation $I_E(t) = J(z_2, E, t)$ (the stationary of I_E being equal to J_0) we find from Eq. (7)

$$J(z_1, E, t) = I_E(t) + \int_{z_1}^{z_2} dz \left(\frac{\partial f_i}{\partial t} - (\kappa - \frac{\partial U}{\partial t})\frac{\partial f_i}{\partial E}\right). \qquad (10)$$

Taking into account that $\partial z_1/\partial E = -1/F(z_1,t)$, $\partial z_1/\partial t = (\partial U/\partial t)/F(z_1,t)$ and using Eqs. (8), (10) we obtain the continuity-like equation that governs the electron motion over the axis of total energy

$$\frac{\partial N}{\partial t} - \frac{\partial}{\partial E}\left(N\left[\kappa - \left\langle\frac{\partial U}{\partial t}\right\rangle\right]\right) = I_{E+W_0}(t) - I_E(t). \qquad (11)$$

Here the angle brackets mean averaging over z

$$\left\langle\frac{\partial U}{\partial t}\right\rangle = \frac{1}{N}\int_{z_1}^{z_2} dz \frac{\partial U}{\partial t} f_i(z,E). \qquad (12)$$

Next we consider the deviations from the stationary solution in the linear approximation. A small periodic over coordinate modulation of the potential $U - U_0 = \delta U_q \exp(-i\omega t + iqz)$ induces the energy dependence of quantities I_E, N in forms $I_E - J_0 = \delta I_q \exp(-i\omega t - iqE/F_0)$, $N - N_0 = \delta N_q \exp(-i\omega t - iqE/F_0)$. We will demonstrate that for $q \approx q_m = \pm 2\pi m/L_0$ (where $m = 1, 2, ..$) the imaginary part of ω is positive which implies that a stationary solution (5) is unstable. For $q = q_m$ the solution is periodic function of energy with a period W_0/m and $I_{E+W_0}(t) = I_E(t)$. Then linearization of Eq. (11) yields

$$\omega_m = \frac{\kappa}{F_0 + \Delta F_m} q_m, \qquad (13)$$

where $\Delta F_m = -iq_m N_0 \langle\delta U_m\rangle / \delta N_m$. We see that the physics of the problem is governed by the only parameter $\langle\delta U_m\rangle / \delta N_m$ (the subscript m implies that all quantities are taken at $q = q_m$). This parameter has a transparent physical meaning of a response of the averaged potential with respect to a small variation of electron density in energy space δN_m. The instability ($\mathrm{Im}(\omega_m) > 0$) occurs, when $Re\,\langle\delta U_m\rangle / \delta N_m > 0$. In order to find this parameter one should go beyond the averaged kinetic equation (11) and solve Eqs. (6), (7) together with Poisson equation. As long as Eq. (13) is already proportional to a small parameter κ, one can simplify the solution of Eqs. (6), (7) assuming that $\kappa = 0$, and neglecting $\partial f_i/\partial t$ and $\partial U/\partial t$ (since Eq. (13) provides that $\omega \sim \kappa$). Then Eq. (7) reduces to $\partial J/\partial z = 0$, which implies that $J(z, E, t) = I_E(t)$. As a result, Eq. (6) yields

$$f_i(z, E, t) = I_E(t) \int_z^{z_2} \frac{dz'}{D(E - U(z', t))}. \qquad (14)$$

The small variation of the distribution function δf_i can be found by linearization of this equation with respect to $\delta I_m, \delta U_m$, the functions z_1 and z_2 being also linearized. The Poisson equation gives us a proportionality between δU_m and the Fourier transform δn_m of the variation of electron concentration

$$\delta n(z,t) = \delta \int_{E_1}^{E_2} dE f_i. \qquad (15)$$

Here the variation includes the variation of δf_i as well as variation of the integration limits $E_1(z,t)$ and $E_2(z,t)$. In this paper we restrict ourselves to the case of 2D semiconductor quantum well, assuming that the dielectric constant ϵ is the same both inside and outside the quantum well. For such structure

$$\delta U_m = \frac{2\pi e^2}{\epsilon |q_m|} \delta n_m. \qquad (16)$$

Using Eqs. (15), (16) and linearized Eqs. (9), (14) one can find the relation between δN_m and δU_m. To calculate the parameter ΔF_m, one should also average the

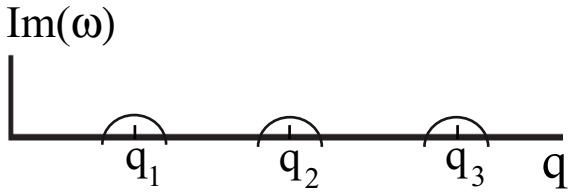

FIG. 2: Instability increment as a function of q. Instability regions correspond to $q \approx q_m = \pm 2\pi m/L_0$

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variation of potential $\delta U_m \exp(iq_m z)$ with the stationary distribution function $f_{i0}(E,z)$ (since we solve the problem in a linear approximation). After cumbersome but rather straightforward calculations finally we get

$$\omega_m = s_0 q_m + i\frac{\kappa}{W_0}\frac{\lambda_m|\alpha_m|^2}{1+\lambda_m\alpha_m}, \qquad (17)$$

where $\alpha_m = \int_0^{L_0} dy(1-\exp(iq_m y))/y$, $\lambda_m = e^2 n_0/m\epsilon F_0$. It is easy to check that for any m, $\mathrm{Im}(\omega_m) > 0$. Thus, for $q = q_m$ a stationary solution is unstable. For a weak field, $\lambda_m \gtrsim 1$, the increment is field independent, $\mathrm{Im}(\omega_m) \sim \kappa/W_0$. One can show that for $q \approx q_m$ the spectrum reads

$$\omega(q) = \omega_m + (q-q_m)v - i\frac{D^*}{4}(q-q_m)^2, \qquad (18)$$

where $D^* = D_0\left(1 + \frac{\lambda_m\alpha_m^*}{1+\lambda_m\alpha_m}\frac{2}{iq_mL_0}\right)$ (we neglected small corrections of the order of κ to v and D^*). This implies that instability exists only in a small vicinity of q_m (see Fig. 2). This instability should lead to the formation of CDW with the periods L_0/m.

Next we discuss a possibility of observation of the effect. The instability increment is proportional to the rate of energy loss κ, which can be calculated for electrons in 2D quantum well in full analogy with the 3D case [2]. For the case of infinitely deep rectangular quantum well of width a, calculations yield

$$\kappa = \frac{C_0^2\pi^2 M}{\rho a^3\hbar}. \qquad (19)$$

Here C_0 is a deformation potential constant, ρ is density of the crystal. This result justifies our assumption that κ does not depend on electron kinetic energy. Also we see that κ rapidly increases with decreasing a. The law $\kappa \sim a^{-3}$ can be understood from simple estimates. The momentum transfer from electron to phonon in the direction perpendicular to the quantum well is of the order of $\hbar/a$. Emission of such a phonon leads to the energy loss $\sim \hbar S/a$, where S is the sound velocity. The energy loss rate by the emission of longitudinal phonons may be neglected due to a small factor $k_{||}a$, where $k_{||}$ is the in-plane wave vector of 2D electron. We find that κ is proportional to the integral over dq_z of the product of energy loss $\hbar S/a$ by the squared matrix element $V_q^2 \sim q \sim 1/a$. The upper limit of the integral is of the same order, of $1/a$, yielding $\kappa \sim a^{-3}$. This implies that the instability is more likely to be observed in thin 2D structures. On the other hand, the instability is suppressed by the electron-electron collisions, which lead to maxwellization of the EDF. Thus, the instability condition is given by $\mathrm{Im}(\omega_m) > 1/\tau_{ee}$, where τ_{ee} is the characteristic time of the electron-electron scattering. Crude estimate of τ_{ee} for hot electrons with characteristic energy W_0 gives $\tau_{ee}^{-1} \sim e^4 n_0/\epsilon^2\hbar W_0$. Having in mind Eqs. (1), (17), one can see that for low electron densities $e^4 n_0/\epsilon^2\hbar < \kappa$, a certain field interval exists, in which the instability can be observed. Simple estimates for GaAs and GaN show that for thin quantum wells, $a \approx 30A^o$, the electron concentration is restricted by small but quite reasonable value $\sim 10^{10} sm^{-2}$.

In conclusion, we have presented a self-consistent theory of kinetic stratification. We have shown that the spatial periods of strata equal to $W_0/F_0 m$ and can be tuned by applied voltage.

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